Linear Algebra I

03/02/2023, Friday, 8:30 – 10:30

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1 Nonsingularity and partitioned matrices (10 + 10 = 20 pts)

Let $A, B \in \mathbb{R}^{n \times n}$. Consider the matrix

$$M = \begin{bmatrix} A & A \\ A & B \end{bmatrix}.$$

a. Show that M is nonsingular if and only if A and $A - B$ are both nonsingular.

b. Suppose that A and $A - B$ are both nonsingular. Find the inverse of M.

2 Determinants (20 pts)

Find the determinant of

$$\begin{bmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a+2)^2 & (b+2)^2 & (c+2)^2 \end{bmatrix}$$

where a, b, c are scalars.

3 Diagonalization (9 + 12 + 9 = 30 pts)

Consider the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}.$$

a. Find its eigenvalues.

b. Is it diagonalizable? (Justify your answer.) If it is, find a diagonalizer.

c. Is it unitarily diagonalizable?

4 Subspaces (5 + 15 = 20 pts)

Let

$$S_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x^2 y = x y^2 \right\}$$

and

$$S_2 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x - y = 0 \text{ and } y + 2z = 0 \right\}.$$

a. Is S_1 a subspace of $\mathbb{R}^2$? If it is, find a basis for S_1 and its dimension.

b. Is S_2 a subspace of $\mathbb{R}^3$? If it is, find a basis for S_2 and its dimension.

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10 pts free